AN ASTROMETRICAL APLANATIC TELESCOPE WITH A FIGURATED FLAT SECONDARY MIRROR

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In 1966 (1) (2) the author pointed out the theoretical possibility of an aplanatic system with a figurated flat secondary. The recent construction of a 61" astrometrical parabolic reflector with a flat secondary at the U.S. Naval Observatory, Flagstaff, Arizona, has made it possible to observe the practical performance of this type of optical systems and has induced the author to investigate the possibilities of improving still further the optics of the system. It was found that such a system could be obtained without difficulties, e.g. a reflector whose focal length would be equal to the existing 61" with coma-free field of 1° of diameter. This would also have the advantage of obtaining more compact and luminous instruments. A model was calculated of a 71" (180 cm.) reflector with a focal length of 1800 cm. ($f = 10\ D_1$).

The first inconveniences of a telescope with a parabolic primary are its third order errors: coma, astigmatism, and field curvature. Although theoretically it has no distortion, the secondary mirror causes such a large obstruction that only those rays form an image that are incident on the primary in a ring at a distance from the optical axis larger than half its diameter. On account of the obstruction caused by the secondary, the comatic image has no vertex and its baricenter is displaced in function of its distance from the optical axis. We should remember that in the parabolics the [illegible] is placed on the sagittal plane and not on the mean focal plane. The image of a parabolic mirror has been studied by various authors (3). Considering the low luminosity of this system, a study with the precision of third order is sufficient if it is also controlled by trigonometrical tracing of some selected rays.

I - The Image produced by a parabolic mirror.

In fig. 1 the X axis coincides with the optical axis of the system. The plane YZ is the entry pupil plane. The plane XY is the meridian plane. The plane XZ is the sagittal plane. u_1 is the angle the principal ray forms with the optical axis on the meridian plane. The object is considered to be at infinity, so that the incident beam is parallel to the principal ray. Y_1 is the ordinate of an incident ray at the entry pupil (meridial plane), Z_1 is the abscissa of an incident ray (sagittal plane).

Equation (1) gives the meridional and sagittal deviations from the image computed with the first order optics on a plane perpendicular to the optical axis in function of the coordinates of the ray at the entry pupil

$$\Delta Y' = -1/2\, Y_1(Y_1^2+Z_1^2)\sum_1^{\kappa}\nu I\nu + 1/2(3Y_1^2+Z_1^2)\,tg\, u_1 \sum_1^{\kappa}\nu II\nu -$$

$$(1) \quad -1/2\, Y_1\, tg^2 u_1 \sum_1^{\kappa}\nu III\nu + 1/2\, tg^3 u_1 \sum_1^{\kappa}\nu V\nu$$

$$\Delta Z' = -1/2\, Z_1(Y_1^2+Z_1^2)\sum_1^{\kappa}\nu I\nu + Y_1 Z_1\, tg\, u_1 \sum_1^{\kappa}\nu II_{\nu} - 1/2\, Z_1\, tg^2 u_1 \sum_1^{\kappa}\nu IV\nu$$

Throughout the whole work, the focal length of the system is taken as the unity ($f = 1$). The sums can be computed either the Seidel (5) or the Burch method (2). In a parabolic system the entry pupil lies on the mirror itself.

$\Sigma_\nu I\nu = 0$ (Spherical aberration); $\Sigma_\nu II\nu = -0{,}500$ (Coma)

$\Sigma_\nu III\nu = +2{,}000$; $\Sigma_\nu IV\nu = 0$

$\Sigma_\nu IIIa\nu = +1{,}000$ (Astigmatism) ; $\Sigma_\nu IVa\nu = +1{,}000$ (Field curvature)

$\Sigma_\nu V\nu = 0$ (Distortion)

$R_t = -\dfrac{1}{\Sigma_\nu III\nu} = -0{,}5$ (Radius of curvature of the tangential focal surface)

$R_s = - - \dfrac{1}{\Sigma_\nu IV_\nu} = \infty$ (Radius of curvature of the sagittal focal surface)

$R_m = -\dfrac{1}{\Sigma_\nu IVa_\nu} = -1{,}000$ (Radius of curvature of the mean focal surface)

The tangential and mean focal surfaces are concave towards the incident light; the sagittal focal surface is flat.

If we take polar coordinates

$$Y_1 = \rho \cos \zeta \;;\; Z_1 = \rho\, sen\, \zeta$$

and substitute the values for the sums, the equation are easyly reduced to:

$$\Delta Y' = -1/4\, \rho^2(2+\cos 2\zeta)\,tg\, u_1 - \rho\cos\zeta\, tg^2 u_1$$

$$\Delta Z'^{(2)} = -1/4\ \rho^2 \text{ sen } 2\zeta \quad tg\ u_1$$

This is a complex figure because an increase of 360° in ζ produces a double increase on the focal plane as can be seen in fig. 2.b ($\rho = D_1/2f$). The serius incovenience of this kind of images is that have no symmetrically defined baricenter and when measuring their coordinates in a measuring device we obtain images which are not easily bisected even with an electronic system. In practice, this incovenience is partially dissimulated by seeing conditions.

II - Images produced by an aplanatic system with a flat secondary

In the aplanatic system we have $\Sigma_\nu I_\nu = \Sigma_\nu II_\nu = 0$. Substituting these values and adopting polar coordinates, we obtain:

$$\Delta Y' = -1/2\ Y_1\ tg^2 u_1\ \Sigma_\nu III_\nu + 1/2\ tg^3 u_1\ \Sigma_\nu V_\nu \quad = -1/2\ \rho \cos\zeta\, tg^2 u_1$$

$$(3)\quad \Sigma_\nu III_\nu + 1/2\ tg^3 u_1 \Sigma_\nu V_\nu$$

$$\Delta Z' = -1/2\ Z_1\ tg^2 u_1\ \Sigma_\nu IV_\nu \quad = -1/2\ \rho \text{ sen } \zeta\, tg^2 u_1\ \Sigma_\nu IV_\nu$$

whence

$$\left(\frac{\Delta Y' - 1/2\ tg^3 u_1\ \Sigma_\nu V_\nu}{1/2\ \rho tg^2 u_1\ \Sigma_\nu III_\nu}\right)^2 + \left(\frac{\Delta Z'}{1/2\ \rho tg^2 u_1\ \Sigma_\nu IV_\nu}\right)^2 = +1$$

i.e. the equation for an ellipse, displaced from its centre on the meridian plane. (Distortion). As we see it is a completely symmetri elliptical image on the tangent focal plane and circular on the mea focal plane. The distortion is negligible as we see in practice. This kind of image with symmetric baricenter is easy to measure. Any systematic effect is avoided when measured with electronic measuring desvices.

III - Computation of the sum values in an aplanat with flat secondary

In this kind of instruments the values of the non-zero sums are a function of the obstruction only, and the desired position of the focal surface gives the obstruction. The position of the focal sur face in a system with a flat secondary (parabolic or aplanatic) is given by the equation:

$$\delta = (2a - 1)\ f$$

and is measured from the vertex of the primary mirror. It is positive if the focal plane lies behind the primary mirror.
Assuming a diameter of the primary mirror of about 71"(180 cm) q must be between $0,52 \leq q \leq 0,55$ in order to obtain reasonable values for δ . As all the parameters in this case only depend on q, they are calulated in table II together with the sum values.
Coefficient of parabolization of the primary:

$$A = \frac{1 + q}{1 - q} = e^2 \quad (e = \text{excentricity of the conic})$$

Coefficient of deformation of flat mirror:

$$B = \frac{-2q}{1 - q}$$

Control Formula: $$A + B = 1$$

Astigmatism: $$\Sigma_\nu IIIa_\nu = \chi = \frac{1 + q}{2q}$$

Field curvature: $$R_m = -q \; ; \; \Sigma_\nu \, IVa_\nu = \frac{1}{q}$$

$$R_s = \frac{-2q}{1 - q} = B \; ; \; \Sigma_\nu IV_\nu = \frac{1 - q}{2q}$$

$$R_t = \frac{-2q}{3 + q} \; ; \; \Sigma_\nu III_\nu = \frac{3 + q}{2q}$$

Distortion: $$\Sigma_\nu V_\nu = \frac{1 - q^2}{2q^2} = \frac{2}{B^2}$$

IV - Spot Diagrams

We have computed spot diagrams for the aplanatic and the parabolic system. Each image is produced by 120 spots computed with equations (2) or (3). Ray tracing was carried out for some beams as a means of control; accordance with the Seidel theory was in the order of 0".01 or better which was to be expected according to the luminosity of the system. Both systems can be compared in fig. 2a and fig. 2b.

V - Final Discussion

The projected instrument can be used simultaneously in various ways:

I - In Astrometry - Putting the plate in a secant plane we obtain images whose diameters are less 0"34 in a field of half a degree of diameter. The tangent plane can be also used. Although the images are small ellipses, because of seeing conditions they appear circular, the same as occurs with small comatic images. Even using a fiel of 40' the images are smaller than 0ʺ85.

II - Photographic Reflector - In this case it is sufficient to bend the plate slightly so that the images will be circles of the order of 1" at the edge of a field of 1°. The sagitta will be only 1.37 mm.

III - Anastigmat - By adding a parafocal plate the remnant astigmatism can be eliminated and the field curvature diminished or eliminated. Fields of 2° of diameter can be obtained with images less than 1". Distortion is larger in this case than in I and II. The field is limited by the diameter of the secondary mirror to a larger degree than by the optical errors of the systems.

IV - Flat-fielded Orthoscopic Anastigmat - With two parafocal plates we can obtain a system practically free of third-order errors. The alternatives III and IV can be resolved rigorously with values somewhat different from the coefficients of parabolization of the primary and secondary. It is however better in this case not to do it and to tolerate a small spherical aberration caused by the introduction of the plates. The values of the figuring strengths seem to be large but in view of the low luminosity of the system the difficulties in the construction are not larger than those in the construction of a parabolic of $f = 6D_1$ ($F = 6$). Combining the mechanical improvements of the 61" with the improved optics in a new, more luminous instrumente $f = 8{,}6\ D_1$ ($F = 8{,}6$) an aplanatic reflector of interesting characteristics could be obtained.

Table I

α	0,52000	0,53000	0,54000	0,55000
δ	0,04f	0,06f	0,08f	0,10f
δ (F=10)	72 cm	108 cm	144 cm	180 cm
δ (F=8,6)	61,92 cm	92,88 cm	123,84 cm	154,80 cm
A	+3,16667	+3,25532	+3.34783	+3,44444
B	-2,16667	-2,25532	-2,34783	-2,44444
$\chi = \Sigma IIIa_\nu$	+1,46154	+1.44340	+1,42593	+1.40909
ΣIII_ν	+3.38462	+3.33019	+3,27778	+3.22727
ΣIV_ν	+0,46154	+0.44340	+0,42593	+0.40909
ΣV_ν	+0,42604	+0.39320	+0.36283	+0.33471
θ'	$17'.71\sqrt{F}$	$17'.82\sqrt{F}$	$17'.93\sqrt{F}$	$18'.04\sqrt{F}$
θ'(F=10)	56'0	56'4	56'7	57'0
θ'(F=8,6)	51'9	52'2	52'6	52'9
α_o(F=10 45')	0,58283	0,59152	0.60021	0.60890
α_o(F=10 60')	0,60378	0,61203	0,62028	0,62854
α_o(F=8.6 45')	0,57404	0,58291	0.59178	0.60066
α_o(F=8.6 60')	0,59205	0.60055	0.60904	0.61754

$\theta = \frac{21'4095}{\sqrt{\chi}}\sqrt{F}$ = Diameter of the field in which the diameter of the stellar image is $\leq 1''$

α = the obstruction ratio when the secondary is just big enough to receive the on-axis pencil.

$$\alpha_o = \alpha + (1-\alpha)\frac{f}{D_1}\,\theta \text{ (Obstruction including field)}$$

$$F = f/D_1 \text{ (Focal ratio)}$$

Table II

SEIDEL SUM

ν	rν	d	Iν	IIν	IIIaν	IVν	IIIν	IVaν	Vν
1	-2.0000		+0.2500	-0.5000	+1.0000	0.0000	+2.0000	+1.0000	0.0000
2	∞	0.4800	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1*			-0.7917	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2*			+0.5417	+0.5000	+0.4615	+0.4615	+1.3846	+0.9231	+0.4260
Σ_ν			0.0000	0.0000	+1.4615	+0.4615	+3.3846	+1.9231	+0.4260

Table III

Equations for the Mirrors

The equations defining the sections of the mirrors are of the type:

$$X_{Ei} = \frac{\gamma_{Ei}^2}{4\, fo_i} + a_i \frac{\gamma_{Ei}^4}{64\, f_{oi}^3} + b_i \frac{\gamma_{Ei}^6}{512\, f_{oi}^5} + \ldots\ldots \quad (i = 1\ ,\ 2)$$

The origin of the coordinates in both cases is taken at the vertex of the respective mirror. This is very convenient from a theoretical point of view but in practice and specially in our case where $fo_2 = \infty$ it is convenient to give the equations another form. In the case of the secondary, the singularity can be taken away in the following manner:

$$\text{If} \qquad \xi = \frac{f_2}{f_1} \quad \text{and if} \qquad a_2 = 1 - e_2^2 = 1 + \frac{\xi^3}{q^4}\, B_2$$

we obtain

$$X_{E_2} = \frac{B\ \gamma_{E2}^4}{64\, q^4\, fo_1^3} + \ldots \qquad (\ \xi \to \infty\ )$$

In practice, it is convenient to take $f_1 = 1$ and to have the vertex of the primary mirror E_1 as the only origin of the coordinates: the equations will then have this form:

Primary : $X_{E1} = A_1\gamma_{E1}^2 + B_1\gamma_{E1}^4 + C_1\gamma_{E1}^6 + D_1\gamma_{E1}^8 + \ldots\ldots$

Secondary: $X_{E2} = d + A_2\gamma_{E2}^2 + B_2\gamma_{E2}^4 + C_2\,\gamma_{E2}^6 + \ldots\ldots$

The coefficients - in the case of a flat secondary - only depend on the obstruction q

$$A_1 = \frac{1}{4}\ ;\ B_1 = -\frac{1}{32}\,\frac{q}{1-q}\ ;\ C_1 = -\frac{1}{384}\,\frac{q}{(1-q)^2}\left[5 - 4q\right]\ ;$$

$$D_1 = -\frac{1}{6144}\,\frac{q}{(1-q)^3}\left[43 - 71q + 30\,q^2\right]$$

$$A_2 = \left(\frac{1-q}{1-q} - 1\right)\frac{1}{4q} = 0\ :\ B_2 = \frac{-1}{32(1-q)\,q^3}$$

for $q = 0.52$ we have

$$X_{E1} = +0.250\,\gamma_{E1}^2 - 0.03385\,4166\,\gamma_{E1}^4 - 0.01716\,2182\,\gamma_{E1}^6 - 0.01086\,1055\,\gamma_{E1}^8 - \ldots$$

$$X_{E2} = 0.4800 - 0.4630\,1893\,68\,\gamma_{E2}^4 - \ldots$$

TESTING EQUATIONS

Subnormal:

$$SN_1 = 8A_1 - 16B_1\,\gamma_{E1}^2 + 2(64B_1^2 - 12C_1)\,\gamma_{E1}^4$$

$$SN_2 = \frac{1}{4B_2\,\gamma_{E2}^2}$$

Caustic:

$$= \frac{1}{2A_i}\,1 + (3 - 192B_i)\frac{\gamma_{Ei}^2}{8} + 3(192B_1^2 - 0.5B_i - 20\,C_i)\,\gamma_{Ei}^4 + \ldots$$

$$= -(1 - 64B_i)\frac{1}{4}\,\gamma_{Ei}^3 - 3(128\,B_i^2 - 16\,C_i)\,\gamma_{E_i}^4 + \ldots$$

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A PRELIMINARY SEARCH OF STARS OF RAPID VARIABILITY

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A T-association in the constellation of the Southern Crown was first studied by Joy (1945). Known members are R CrA, T CrA, S CrA and TY CrA which present high peculiarities in both their spectra and their colors (for more details see Joy, 1945; Mendoza, 1968 and 1969; and Mendoza and the Jaschek's, 1968).

This work gives preliminary results of a search of stars of rapid variability in brightness in the neighborhood of NGC 6729. This program will be extended to other centers of the Southern Hemisphere.

Six plates were secured with the Curtis Schmidt Telescope of the Cerro Tololo Inter-American Observatory on September 1968. The plates cover an area of twenty-five square degrees. We used the 103a-0 emulsion behind an ultraviolet filter, UG5. Each plate is composed of several images; the first two are 0.14 mm apart and the remaining are separated only 0.10 mm. The number of images are from five to seven, each one of 15 minutes exposure.

In these twenty-five square degrees are many known variables (Kukarkin, Parenago, Efremov, and Kholopov, 1958); however, we found two stars not listed as variables which had an increase in brightness of nearly two magnitudes in less than two hours. These stars are listed in Table 1. The columns of this Table give, first our number; second, the 1950.0 coordinates (Boss et al, 1937); third, an approximate photographic magnitude at minimun light; fourth, the date (JD) of the maximum; and last, the total estimate duration of the event.

T A B L E 1

TWO RAPID VARIABLES

Star	α (1950.0)	δ	m_{ph}	J D	Δt (min)
1	$18^h\ 54^m\ 08^s$	-36° 38!6	13,6	2440114.518	90
2	18 57 54	37 00.7	18	2440114.550	60

Identification charts for stars listed in Table 1 are given Figures 1 and 2 (North is at the top, East to the left).

Variable 1 is located approximately half a degree to the West of the globular cluster NGC 6723. Thus, it is probably too far and too bright to be a part of this cluster. On the other hand, the known T Tauri-like objects of the association are not close enough to affirm that star 1 belongs to the T-association. However, an infrared plate (IN + W89b), taken on September 15.15, 1968 (UT), indicates a color index K type-like star. Most stars closer than one

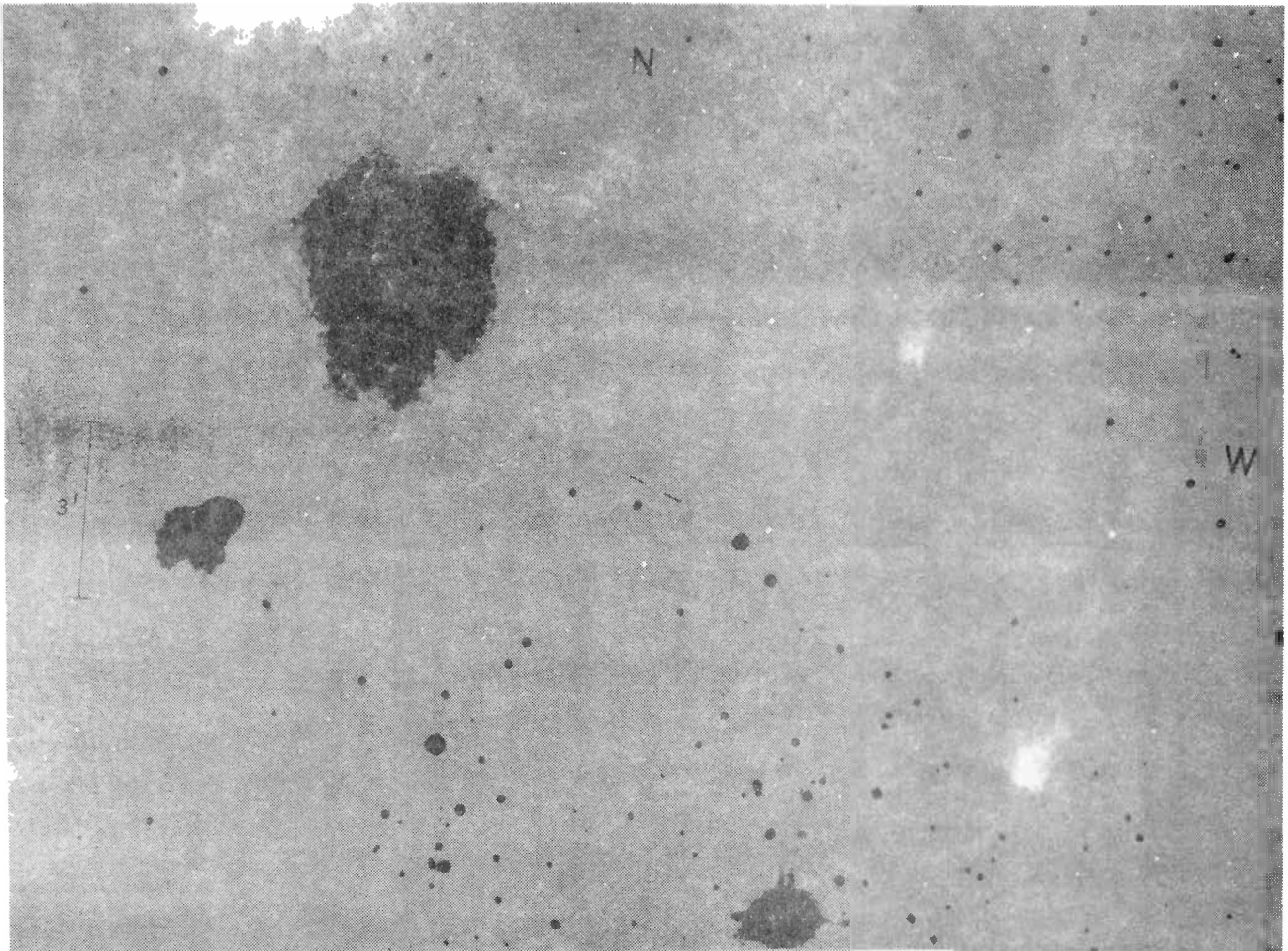
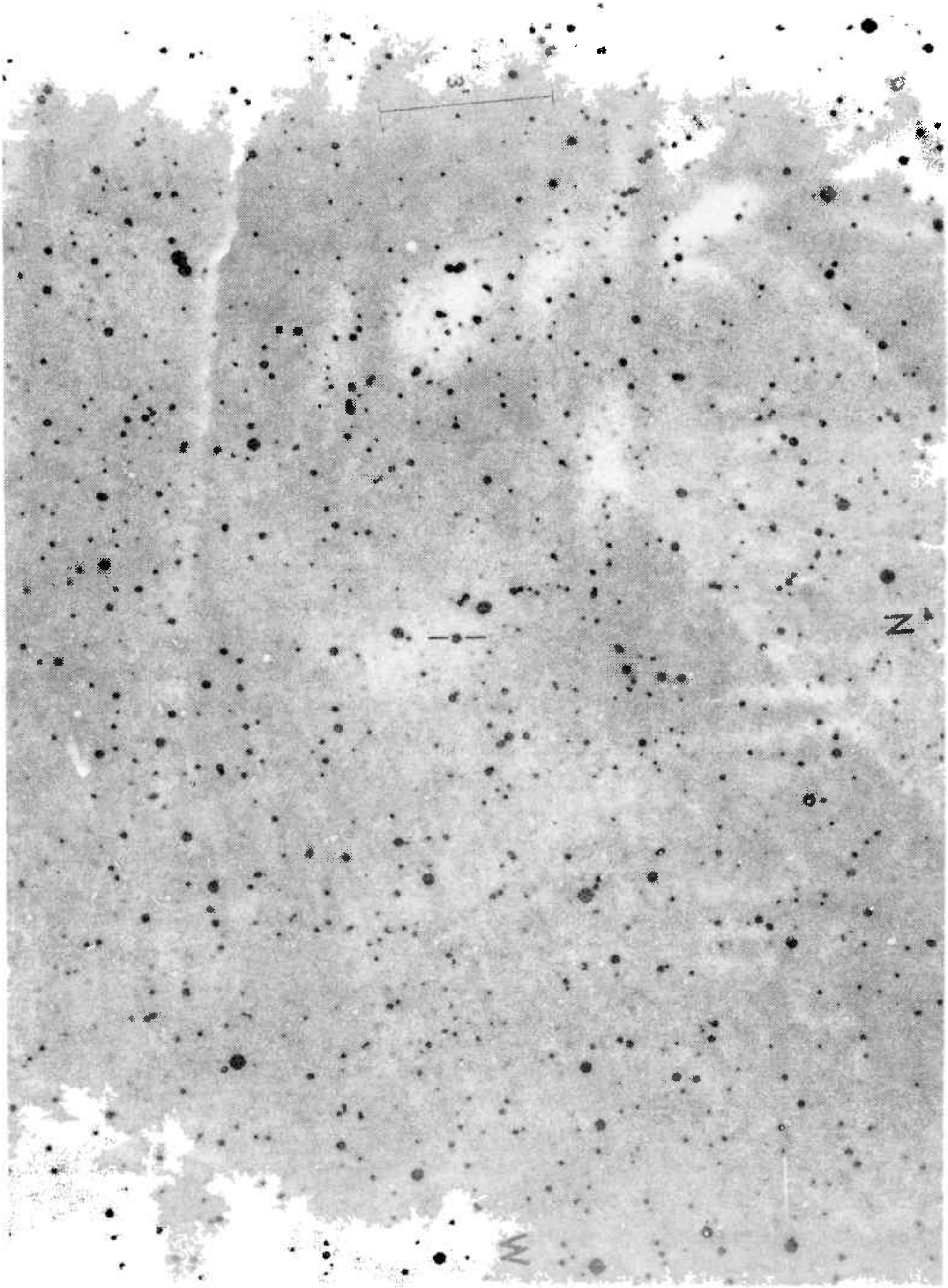


Fig. 2. Field of variable 2 in blue light.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minute of arc are much bluer than variable 1. Some of these stars (see Fig.1) very nicely shape a horseshoe. The area in general does not seem much affected by interstellar extinction.

Variable 2 is located very close to S CrA; thus, it appears likely that it belongs to the association. The infrared color-index seems bluer than that of variable 1. Therefore, the spectral type probably is earlier than of star 1. Star 2 maybe affected by interstellar extinction.

The telescope was used according to an agreement between AURA, Inc. and the University of Chile. We express our thanks to Dr. V.M. Blanco for all the facilities granted to us in Tololo.

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ERRORES SISTEMATICOS DE LOS CATALOGOS FK4 y N_{30}

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Se presentan los resultados de 147 series de observaciones de estrellas fundamentales efectuadas entre las declinaciones -40° y -90° en culminación superior y -90° a -69° en culminación inferior, con el Círculo Meridiano Repsold del Observatorio Astronómico Nacional.

Las reducciones de las observaciones se realizaron con el computador IBM 360 de la Universidad de Chile y en los resultados se incluyen 535 valores de $\Delta\alpha$ y 1494 valores de Δ n.

Los resultados de las observaciones demuestran que el sistema del instrumento está más de acuerdo con el catálogo FK4, que con el N_{30}.

El artículo será publicado 'in extenso' en las Publicaciones del Departamento de Astronomía de la Universidad de Chile.

EFECTO DE NUBES OSCURAS ARTIFICIALES SOBRE RECUENTOS ESTELARES PROMEDIOS

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Justamente hace 10 años, en la 1a. Reunión de la Asociación Astronómica Argentina, aquí en San Juan en 1958, el autor habló sobre el 1er. capítulo de sus investigaciones basadas sobre el análisis de los recuentos estelares promedios en todas las latitudes galácticas, observados y publicados en 1925 por Seares, van Rhijn, Joyner y Richmond. Estos análisis se efectuaron aplicando el método de Bok (1931) cuya base es el Esquema Kapteyn. De esta manera, como resultado fundamental, habían sido perfeccionados cinco Esquemas Kapteyn, es decir cinco curvas Wolf de recuentos estelares promedios, simultáneamente para las cinco latitudes galácticas típicas: $|B| = 0^\circ; 1\overset{\circ}{.}8: 5\overset{\circ}{.}7\ 18\overset{\circ}{.}4 : 90^\circ$.

Este material enorme de cifras, quedando ahora a nuestra disposición y preparado óptimamente según nuevos puntos de vista, resultó un reto para aprovechar algo más este material, dando origen así para el 2do. capítulo de nuestras investigaciones de esta índole. Nada fue más fácil ahora que dejar imprimir sus efectos una serie de nubes absorbentes artificiales según un determinado plan, para ver como se modificarán entónces las curvas Wolf. Tal colección sistemática de curvas de recuentos estelares promediados sin y con influencia de determinadas pantallas de absorción interestelar debería ser capaz de dar también en el futuro indicios valiosos del posible poder y distancia de tales nubes absorbentes en casos formados especialmente en la realidad.